Summary. The Common-Reflection-Surface (CRS) stack is a fully-automated, data-driven stacking method, i.e., a knowledge of a macro-velocity model is not required except from near-surface velocities at the sources and receivers. It is well-known that the CRS stack for common-offset (CO), i.e., the prestack data are stacked into a CO gather rather than into a zero-offset (ZO) section, is also able to handle converted waves (Bergler et al., 2002) which is of particular interest for land seismics, vertical seismic profiling (VSP) and ocean bottom seismics (OBS). A new hyperbolic traveltime formula for the 2D Common-Offset Common-Reflection-Surface (2D CO CRS) stack that takes arbitrary top-surface topography into account is presented. This formula can then be used to derive stacking operators that are in principle able to handle a VSP acquisition geometry as well as reverse VSP and cross-well seismsics. Moreover, the application of the 2D CO CRS stack to OBS acquisition geometries is discussed.

Introduction. The 2D CRS stack was originally developed to stack prestack data acquired along a straight line on a planar measurement surface into a ZO section (2D ZO CRS stack) (Mann et al., 1999; Müller, 1999). The stacking operator describes a surface rather than a curve and depends on so-called kinematic wavefield attributes. The attributes that yield the stacking operator that fits best an actual event in the prestack data is determined by means of coherence analysis. Zhang et al. (2001) extended the CRS stack for finite-offset (FO), i.e., the prestack data are stacked into a FO gather, e.g., into a CO gather (2D CO CRS stack, Bergler, 2001). Both authors considered only planar measurement surfaces. Zhang (2003) derived the most general moveout formula used in the CRS stack to handle

- 3D data acquisition on a measurement surface
- arbitrary top-surface topography, source and receiver elevations are explicitely considered
- velocity gradients in the vicinity of the sources and receivers.

Zhang (2003) used this formula in order to derive stacking operators for the 2D ZO CRS stack in the presence of arbitrary topography and for the 2D CO CRS stack for planar measurement surfaces. We present a 2D CO CRS stack traveltime formula for arbitrary top-surface topography. We will not consider velocity gradients.

Arbitrary topography. The traveltime formula for the 2D CO CRS stack for arbitrary topography can directly be derived from the general moveout formula given in Zhang (2003) by setting

- the azimuth angles at the sources and receivers equal zero, i.e., $\theta_s = 0$ and $\theta_G = 0$,
- all the variables associated with the y-direction equal zero,
- and the gradients of the near surface velocity equal zero, i.e., $\nabla v(S) = 0$ and $\nabla v(G) = 0$.

The first two items are due to the fact that we consider 2D data acquisition along a straight line. Putting these assumptions into the general traveltime equation yields the searched-for hyperbolic traveltime formula:

$$T^2(\Delta x_S, \Delta x_G, \Delta z_S, \Delta z_G) = \left( t_0 + \frac{\sin \beta_G}{v_G} \Delta x_G - \frac{\sin \beta_S}{v_S} \Delta x_S + \frac{\cos \beta_G}{v_G} \Delta z_G - \frac{\cos \beta_S}{v_S} \Delta z_S \right)^2 + t_0 DB^{-1} (\Delta x_G - \Delta z_G \tan \beta_G)^2$$
$$+ t_0 AB^{-1} (\Delta x_S - \Delta z_S \tan \beta_S)^2 - 2t_0 B^{-1} (\Delta x_G - \Delta z_G \tan \beta_G) (\Delta x_S - \Delta z_S \tan \beta_S).$$

(1)
A, B, and D are three elements of the surface-to-surface ray propagator matrix in a global coordinate system. $t_0$ denotes the traveltime along the central ray, $v_s$ and $v_g$ are the near-surface velocities at the source and receiver. $\beta_S$ and $\beta_G$ are the emergence angles of the central ray at source and receiver, respectively. $\Delta x_S$ and $\Delta x_G$ denote the horizontal displacement between sources and receivers of central and paraxial ray. $\Delta z_S$ and $\Delta z_G$ are the corresponding vertical displacements. Note that Equation (1) is given in a global coordinate system, whereas Zhang (2003) used a ray-centered coordinate system. Thus, we applied a transformation from ray-centered into global coordinates. Zhang et al. (2001) and Bergler (2001) related the elements $A$, $B$, $C$, and $D$ of the ray propagator matrix to three wavefront curvatures $K_1$, $K_2$, and $K_3$, where $K_1$ is defined as the wavefront curvature of an emerging wave at the receiver in a common-shot (CS) experiment, while $K_2$ and $K_3$ are the wavefront curvatures at the source and receiver, respectively, in a (hypothetical) common-midpoint (CMP) experiment. It is straightforward to express Equation (1) in terms of wavefront curvatures to allow a geometrical interpretation of all five wavefield attributes. With $\Delta z_S = \Delta z_G \equiv 0$, we obtain the original 2D CO CRS stacking operator for a planar measurement surface as given in Zhang et al. (2001).

Ocean bottom seismics. Figure 1 shows a simple sketch of a typical 2D OBS acquisition geometry with receivers on the ocean bottom and sources some meters below the water surface. We assume a virtually horizontal ocean bottom without significant topography and a constant source depth. This implies $\Delta z_S = \Delta z_G \equiv 0$, see also Figure 1. Thus, we may use the original 2D CO CRS stacking operator developed to stack data acquired along one straight line on a planar measurement surface (e.g., land seismics or conventional marine data acquisition). This operator can be written as (Bergler, 2001; Zhang et al., 2001)

$$T^2(\Delta x_S, \Delta x_G) = \left( -t_0 \frac{\sin \beta_S}{v_G} \Delta x_S - \frac{\sin \beta_S}{v_S} \Delta z_S \right)^2 + t_0 (DB^{-1} \Delta z_G^2 + AB^{-1} \Delta x_S^2 - 2B^{-1} \Delta x_G \Delta x_S),$$  
 expresses in source and receiver dislocations $\Delta x_S$ and $\Delta x_G$, respectively. If there is significant topography present at the ocean bottom, it is possible to use Equation (1) to take the topography into account. This does not affect the simplifying assumption $\Delta z_S \equiv 0$.

Vertical seismic profiling. Equation (1) can be used to derive stacking operators applicable to VSP data. A typical 2D VSP acquisition geometry is characterized by receivers placed in a borehole while the sources are located along a straight line on the top-surface (Figure 1). VSP is also possible in marine environments. In this case, the sources are located some meters below the water surface. Let us assume a vertical borehole and that all sources are disposed at the same level, i.e., in the same water depth or on a measurement surface on land without topography: the vertical displacements between the sources $\Delta z_S$ and the horizontal displacements between the receivers $\Delta x_G$ vanish, i.e., $\Delta z_S = \Delta x_G \equiv 0$. Thus, for such VSP geometries, Equation (1) simplifies to

$$T^2(\Delta x_S, \Delta z_G) = \left( -t_0 \frac{\sin \beta_S}{v_S} \Delta x_S + \frac{\cos \beta_G}{v_G} \Delta z_G \right)^2 + t_0 (DB^{-1} \Delta z_G^2 + AB^{-1} \Delta x_S^2 + 2B^{-1} \tan \beta_G \Delta z_G \Delta x_S).$$  

Figure 1: An arbitrarily chosen central ray and a paraxial ray in the close vicinity of the central ray for a 2D OBS (left) and VSP (right) acquisition geometry.
In principle, this operator has the same structure as the original 2D CO CRS stacking operator and simplifies in different subsets of the prestack data volume where the stacking surface reduces to a stacking curve. Thus, similar pragmatic search strategies can be applied. Furthermore, stacking operators for so-called reverse VSP and cross-well acquisition geometries can easily be derived by means of Equation (1). In the former case, the sources are placed downhole while the receivers are deployed at the surface. Thus, $\Delta x_S = \Delta z_G \equiv 0$, assuming a vertical borehole and a planar and horizontal measurement surface. Cross-well acquisition means that both, sources and receivers, are placed downhole in neighboring boreholes. In this case, $\Delta x_S = \Delta x_G \equiv 0$, assuming vertical boreholes.

**OBS data example.** To demonstrate the ability to process OBS data, we applied the 2D CO CRS stack to a synthetic OBS data set. The model (Figure 2) consists of four layers separated by curved interfaces. Shot spacing is 25 m, receiver spacing 50 m. The modeled prestack data contains the primary PP reflections with a zero-phase Ricker wavelet, peak frequency 30 Hz, and a sampling interval of 4 ms. The aspects of multi-component data are discussed in a separate contribution. Figure 3 shows a CO section extracted from the prestack data, the CRS-stacked section, and one of the wavefield attribute sections. All five data-derived wavefield attributes are in good agreement with their forward-modeled counterparts (not shown).

**Conclusions.** We presented a new hyperbolic 2D CO CRS stack traveltime formula to handle arbitrary top-surface topography. We observed that this formula reduces to the original one for an OBS acquisition geometry if there is no significant topography present at the ocean bottom. Moreover, the formula was specialized for VSP acquisition geometries with vertical boreholes. Stacking operators for reverse VSP as well as for cross-well seismics can also be derived. Our approach is of particular interest in combination with multi-component data. The application of the operator was successfully demonstrated with a synthetic OBS data set.

**Acknowledgments**

We would like to thank the sponsors of the *Wave Inversion Technology Consortium* for their support.

**References**


Figure 3: CO section for $h = 500$ m extracted from the prestack data (top). CO CRS-stacked section (middle) and the associated $\beta_G$ section (bottom).