Data-driven imaging with second-order traveltime approximations

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Overview

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- Outlook
Motivation

Model-based approaches:

- Sensitive to model errors
- Migration velocity analysis is costly

Data-driven approaches:

- Interval velocity model determination is postponed
- Robust methods
- However, classic data-driven approaches use only a subset of available data, thus no optimum S/N ratio
- Provide little information for later inversion
- Data-driven aspects usually not fully exploited
Motivation

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Common-Reflection-Surface (CRS) stack:
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- minimum a priori information required
Motivation

Common-Reflection-Surface (CRS) stack:

- extension of concepts of classic data-driven approaches
- full use of available data
- minimum a priori information required
- fully data-driven application
Data example A

2-D NMO/DMO/stack – from Müller (1999)

CMP location approx. 13km
Data example A

2-D CRS stack – from Müller (1999)
Data example A

NMO/DMO/stack/poststack migration – from Müller (1999)
Data example A

2-D CRS/poststack migration – from Müller (1999)
Data example B

NMO/DMO/stack vs. CRS stack – 3-D data, inline
From Bergler et. al (2002). Data courtesy of ENI E & P Division.
Data example C

Conventional 3-D prestack depth migration

Courtesy of ENI E & P Division
Data example C

3-D poststack depth migration of CRS stack
Courtesy of ENI E & P Division
Data example C

depth slices of coherence images: conventional vs. CRS-based

Courtesy of ENI E & P Division
Basic concepts

Derive an approximation of the kinematic reflection response for a reflector segment in depth characterized by its local dip and local curvature, i.e., the reflector properties up to second order. Use parameters defined either in the time domain — traveltime derivatives — or in the depth domain at the acquisition surface — properties of hypothetical wavefronts, both linked by the near-surface velocity $v_0$. 
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    ➔ traveltime derivatives
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- Determine optimum stacking operator by means of coherence analysis in the data.
  ➥ generalized multi-dimensional velocity analysis
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- Stack along the determined stacking operator.
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Results:
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Results:

- a simulated section for an arbitrarily chosen configuration
- a set of associated wavefield attribute sections
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Results:

- a simulated section for an arbitrarily chosen configuration
- a set of associated wavefield attribute sections
  ➡ subsequent applications like velocity determination
- an associated coherence section
  ➡ identification of events, reliability of attributes
Derivation

Possible ways to derive an approximation of the kinematic reflection response:
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- paraxial ray theory, i.e., assumption of a linear relation between the properties of neighboring rays
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- geometrical optics using the concept of object and image points (2-D case only)
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- paraxial ray theory, i.e., assumption of a linear relation between the properties of neighboring rays
- geometrical optics using the concept of object and image points (2-D case only)
- pragmatic way: second-order expansion of traveltime, initially without physical interpretation
Derivation

Prestack data:

(hyper-)volume $p(t, \vec{m}, \vec{h})$ with up to five dimensions
Derivation

Prestack data:

\( p(t, \vec{m}, \vec{h}) \) with up to five dimensions

\[
\begin{align*}
t &\quad \text{time} \\
\vec{m} &= \begin{pmatrix} m_x \\ m_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x + s_x \\ g_y + s_y \end{pmatrix} \\
\vec{h} &= \begin{pmatrix} h_x \\ h_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x - s_x \\ g_y - s_y \end{pmatrix}
\end{align*}
\]

midpoint vector

half-offset vector
Derivation

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(hyper-)volume \( p(t, \vec{m}, \vec{h}) \) with up to five dimensions

\[
\begin{align*}
    t & \\
    \vec{m} &= \begin{pmatrix} m_x \\ m_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x + s_x \\ g_y + s_y \end{pmatrix} & \text{time} \\
    \vec{h} &= \begin{pmatrix} h_x \\ h_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x - s_x \\ g_y - s_y \end{pmatrix} & \text{midpoint vector} \\
\end{align*}
\]

Reflection event:

(hyper-)surface \( t \left( \vec{m}, \vec{h} \right) \) in the prestack data
Central and paraxial rays

Assumed to be known: traveltime $t\left(\vec{m}, \vec{h}\right)$ along central ray (SRG)

$$\Delta \vec{h} = \vec{h}^* - \vec{h}$$
Central and paraxial rays

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How to approximate \( t \left( \vec{m} + \Delta \vec{m}, \vec{h} + \Delta \vec{h} \right) \) along paraxial ray (S*R*G*)?

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Pragmatic approach

Taylor expansion up to second order – general case
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\[ t \left( \tilde{m} + \Delta \tilde{m}, \tilde{h} + \Delta \tilde{h} \right) \approx \]
Pragmatic approach

Taylor expansion up to second order – general case

\[ t \left( \vec{m} + \Delta \vec{m}, \vec{h} + \Delta \vec{h} \right) \approx t \left( \vec{m}, \vec{h} \right) \]
Pragmatic approach

Taylor expansion up to second order – general case

\[
\begin{align*}
t \left( \vec{m} + \Delta \vec{m}, \vec{h} + \Delta \vec{h} \right) & \approx \\
t \left( \vec{m}, \vec{h} \right) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y
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\quad + \frac{1}{2} \left( \frac{\partial^2 t}{\partial m_x^2} \Delta m_x^2 + \frac{\partial^2 t}{\partial m_y^2} \Delta m_y^2 + \frac{\partial^2 t}{\partial h_x^2} \Delta h_x^2 + \frac{\partial^2 t}{\partial h_y^2} \Delta h_y^2 \right)
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Taylor expansion up to second order – general case

\[ t \left( \bar{m} + \Delta \bar{m}, \bar{h} + \Delta \bar{h} \right) \approx \]

\[ t ( \bar{m}, \bar{h} ) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y \]

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Pragmatic approach

Special case: Marine acquisition, single azimuth

\[ t \left( \vec{m} + \Delta \vec{m}, \vec{h} + \Delta \vec{h} \right) \approx \]

\[ t \left( \vec{m}, \vec{h} \right) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y \]

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Pragmatic approach

Special case: 2-D acquisition

\[ t \left( \tilde{m} + \Delta \tilde{m}, \tilde{h} + \Delta \tilde{h} \right) \approx \]

\[ t \left( \tilde{m}, \tilde{h} \right) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y \]

\[ + \frac{1}{2} \left( \frac{\partial^2 t}{\partial m_x^2} \Delta m_x^2 + \frac{\partial^2 t}{\partial m_y^2} \Delta m_y^2 + \frac{\partial^2 t}{\partial h_x^2} \Delta h_x^2 + \frac{\partial^2 t}{\partial h_y^2} \Delta h_y^2 \right) \]

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Pragmatic approach

General case

\[ t \left( \tilde{m} + \Delta \tilde{m}, \tilde{h} + \Delta \tilde{h} \right) \approx \]

\[ t \left( \tilde{m}, \tilde{h} \right) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y \]

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Pragmatic approach

Special case: zero-offset simulation

\[
t \left( \tilde{m} + \Delta \tilde{m}, \tilde{h} + \Delta \tilde{h} \right) \approx \\
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Pragmatic approach

Special case: zero-offset simulation, marine case

\[
t \left( \vec{m} + \Delta \vec{m}, \vec{h} + \Delta \vec{h} \right) \approx \\
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Pragmatic approach

Special case: zero-offset simulation, 2-D acquisition

\[ t \left( \tilde{m} + \Delta \tilde{m}, \tilde{h} + \Delta \tilde{h} \right) \approx \]

\[ \begin{align*}
  & t \left( \tilde{m}, \tilde{h} \right) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y \\
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\end{align*} \]
Pragmatic approach

Special case: ZO simulation, 2-D, CMP gathers only

\[ t \left( \tilde{m} + \Delta \tilde{m}, \tilde{h} + \Delta \tilde{h} \right) \approx \]

\[ t \left( \tilde{m}, \tilde{h} \right) + \frac{\partial t}{\partial m_x} \Delta m_x + \frac{\partial t}{\partial m_y} \Delta m_y + \frac{\partial t}{\partial h_x} \Delta h_x + \frac{\partial t}{\partial h_y} \Delta h_y \]

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Pragmatic approach

Preliminary conclusions:

In some cases, not all derivatives are independent in the context of paraxial ray theory. This is not evident at this stage! Hyperbolic approximations can be obtained by squaring and neglecting higher order terms. We need a physical interpretation of the derivatives to identify hidden dependencies, to understand which values are physically reasonable, and to make use of the derivatives for various purposes.
Preliminary conclusions:

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- Hyperbolic approximations can be obtained by squaring and neglecting higher order terms.

- We need a physical interpretation of the derivatives to identify hidden dependencies,
  to understand which values are physically reasonable,
  and to make use of the derivatives for various purposes.
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]
Physical interpretation

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\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

Horizontal slowness:

\[ p_x = \left. \frac{1}{2} \frac{\partial t}{\partial x_m} \right|_{(x_m=x_0, h=0)} \]
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[
t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right]
\]

Horizontal slowness:

\[
p_x = \frac{1}{2} \frac{\partial t}{\partial x_m} \bigg|_{x_m=x_0, h=0} = |\vec{p}| \sin \alpha
\]

\(\vec{p}\) slowness vector
\(\alpha\) emergence angle
\(v_0\) near-surface velocity
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Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

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- \( \vec{p} \) slowness vector
- \( \alpha \) emergence angle
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Curvature of “zero-offset wavefront”:

\[ K_N = \left. \frac{\partial^2 t}{\partial x_m^2} \right|_{(x_m = x_0, h = 0)} \]
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

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\[ K_N = \frac{v_0}{2} \left. \frac{\partial^2 t}{\partial x_m^2} \right|_{(x_m=x_0, h=0)} \]
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Curvature of “zero-offset wavefront”:

\[ K_N = \frac{v_0}{2} \frac{1}{\cos^2 \alpha} \frac{\partial^2 t}{\partial x_m^2} \bigg|_{(x_m = x_0, h = 0)} \]
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

Curvature of “zero-offset wavefront”:

\[ K_N = \frac{v_0}{2} \frac{1}{\cos^2 \alpha} \left. \frac{\partial^2 t}{\partial x_m^2} \right|_{x_m=x_0, h=0} \]

A “zero-offset wavefront”, also called normal wavefront, can be obtained from an exploding reflector experiment.
Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]
Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

Curvature of “common-midpoint (CMP) wavefront”: 
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

Curvature of “common-midpoint (CMP) wavefront”:

Problem: no simple physical experiment available!
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

Curvature of “common-midpoint (CMP) wavefront”:

Problem: no simple physical experiment available!

However: up to second order, CMP traveltimes and zero-offset diffraction traveltimes coincide (NIP wave theorem, Hubral 1983).
Physical interpretation

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Problem: no simple physical experiment available!

However: up to second order, CMP traveltimes and zero-offset diffraction traveltimes coincide (NIP wave theorem, Hubral 1983).

In analogy to the exploding reflector experiment, a exploding reflection point experiment approximates the “CMP wavefront”.
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

Curvature of “common-midpoint (CMP) wavefront”:

\[ K_{NIP} = \frac{1}{2} \frac{v_0}{\cos^2 \alpha} \frac{\partial^2 t}{\partial h^2} \bigg|_{(x_m=x_0, h=0)} \]
Physical interpretation

Simplest case: 2-D acquisition, zero-offset

\[ t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m} (x_m - x_0) + \frac{1}{2} \left[ \frac{\partial^2 t}{\partial x_m^2} (x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2 \right] \]

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An exploding reflection-point experiment yields the so-called normal-incidence-point (NIP) wavefront.
Physical interpretation

Replacing all derivatives, we obtain

\[ t(x_m, h) = t_0 + \frac{2 \sin \alpha}{v_0} (x_m - x_0) + \frac{\cos^2 \alpha}{v_0} \left[ K_N (x_m - x_0) + K_{NIP} h^2 \right] \]

in terms of *kinematic wavefield attributes*. 
Replacing all derivatives, we obtain

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in terms of *kinematic wavefield attributes*. Accordingly, the hyperbolic counterpart reads

\[ t^2(x_m, h) \approx \tilde{t}^2(x_m, h) = \left[ t_0 + \frac{2 \sin \alpha}{v_0} (x_m - x_0) \right]^2 \]

\[ + \frac{2 t_0 \cos^2 \alpha}{v_0} \left[ K_N (x_m - x_0)^2 + K_{NIP} h^2 \right]. \]
Data example A

2-D CRS stack – from Müller (1999)
Data example A

Emergence angle $\alpha \, [^\circ]$
Data example A

Radius of curvature of NIP wavefront [m]
Data example A

Radius of curvature of normal wavefront [m]
Applications of attributes

Construction of interval velocity models based on picked zero-offset traveltimes and attributes with a generalized Dix-type inversion: layer stripping approach downward propagation of NIP wavefronts until \( R_{\text{NIP}} = 0 \) \( t_0 = 0 \) a tomographic approach: initial model of interval velocity and reflector segments forward modeling of NIP wavefronts iterative model updates to minimize misfit
Applications of attributes

- Construction of interval velocity models based on picked zero-offset traveltimes and attributes with
Applications of attributes

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Construction of interval velocity models based on picked zero-offset travel times and attributes with a generalized Dix-type inversion:
- layer stripping approach

\[ R_{NIP} = 0 \]
\[ t_0 = 0 \]
Applications of attributes

- Construction of interval velocity models based on picked zero-offset traveltimes and attributes with a generalized Dix-type inversion:
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Applications of attributes

Construction of interval velocity models based on picked zero-offset traveltimes and attributes with

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Applications of attributes

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Reconstructed vs. original model

Reconstructed velocity and reflector elements

Original velocity and reconstructed reflector elements
Applications of attributes

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☞ oral presentation on Wednesday afternoon ☞
Applications of attributes

Based on approximation of diffraction traveltimes:
- approximation of geometrical spreading factor
- approximation of projected Fresnel zone
- data-driven time migration
- identification of diffraction events

Based on moveout-corrected CRS super gathers:
- residual statics correction
Applications of attributes

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Applications of attributes

Extensions based on attribute extrapolation at surface:
- CRS stack for smooth topography considers dip and curvature of acquisition surface with the same traveltime formula as without topography.
- CRS stack for rugged topography uses direct use of source and receiver elevations, treating wavefield attributes as if recorded on a plane surface.

Redatuming
Applications of attributes

Extensions based on attribute extrapolation at surface:

- CRS stack for smooth topography
  - considers dip and curvature of acquisition surface
  - same traveltime formula as without topography
- CRS stack for rugged topography
  - direct use of source and receiver elevations
  - wavefield attributes as if recorded on plane surface
- Redatuming
Applications of attributes

Extensions based on attribute extrapolation at surface:

- CRS stack for smooth topography

poster presentation on Tuesday afternoon
Applications of attributes

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- Redatuming
Applications of attributes

Synthetic example with topography

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Applications of attributes

Optimized CRS stack
Applications of attributes

Redatumed CRS stack section
Conclusions

- consequent generalization of classic data-driven approaches
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- requires minimum interaction
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- CRS stack
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  - attribute-based velocity determination
Conclusions

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  - CRS stack
  - Attribute-based velocity determination
  - Poststack migration of CRS result and/or
Conclusions

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- Requires minimum interaction
- Provides wavefield attributes for various applications
- Allows consistent processing workflow
  - CRS stack
  - Attribute-based velocity determination
  - Poststack migration of CRS result and/or
  - Prestack migration based on inversion result
Outlook

- implementation of 3-D inversion (in progress)
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- data regularization
Acknowledgments

This work was supported by the sponsors of the *Wave Inversion Technology Consortium*.